AP Calculus BC Summer Work

Google Classroom

class code: ifsxqfr

Calculus - SUMMER PACKET

NAME:_____

Summer + Math = (Best Summer Ever)^2

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

Use the graph $f(x)$ to answer the following.

4. $f(0) =$

$f(4) =$

$f(-1) =$

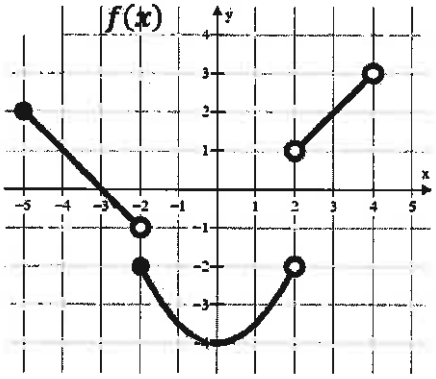
$f(-2) =$

$f(2) =$

$f(3) =$

$f(x) = 2$ when $x =$?

$f(x) = -3$ when $x =$?



Write the equation of the line meets the following conditions. Use point-slope form.
$y - y_1 = m(x - x_1)$

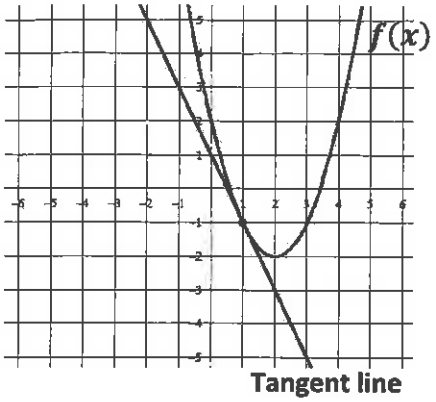
5. slope $= 3$ and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

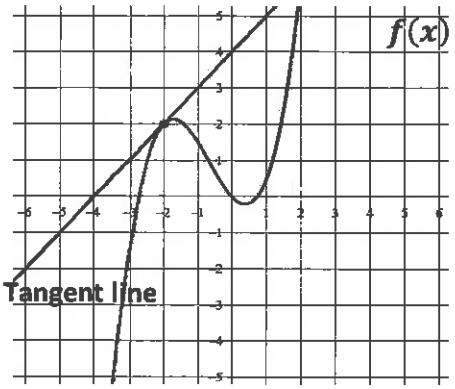
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$

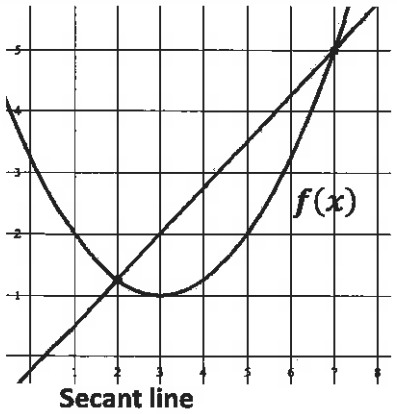


9. The line tangent to $f(x)$ at $x = -2$



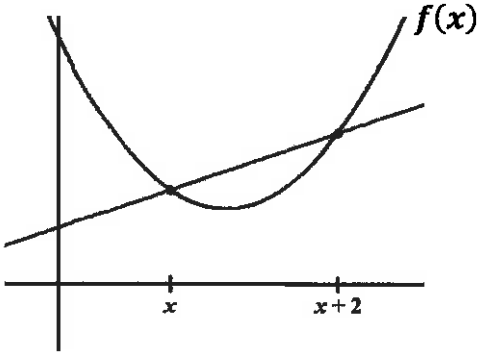
MULTIPLE CHOICE! Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?



A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

11. Which choice represents the slope of the secant line shown?



Secant line

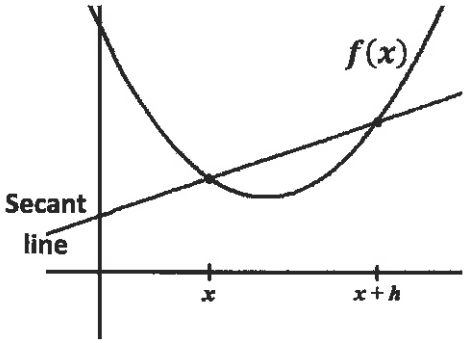
A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

12. Which choice represents the slope of the secant line shown?

A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

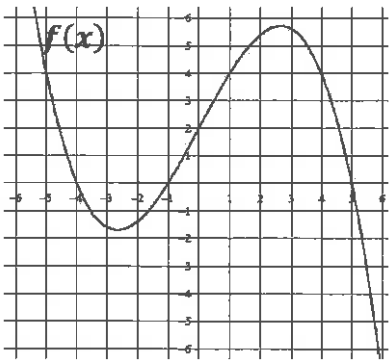
D) $\frac{f(x)-f(x+h)}{x+h-x}$



13. Which of the following statements about the function $f(x)$ is true?

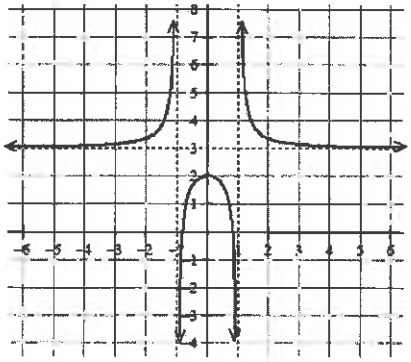
I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only



Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



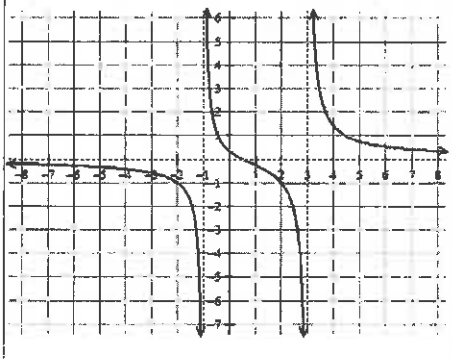
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



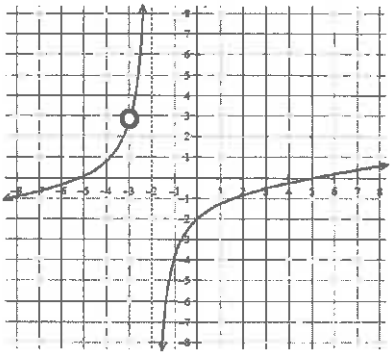
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.

31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.

46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \leq x \leq 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	Domain	Range
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

76. $\ln \frac{1}{2}$

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $f(x) = \{(3,5),(2,4),(1,7)\}$ $g(x) = \sqrt{x-3}$, $h(x) = \{(3,2),(4,3),(1,6)\}$ $k(x) = x^2 + 5$ **, then determine each of the following.**

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f(h(3))$

84. $g(k(7))$

85. $h(3)$

86. $g(g(9))$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k(g(x))$

90. $g(f(2))$

PRECALCULUS-CALCULUS READINESS EXAMINATION

Test # 2

1. When $a = 2$, $b = -3$, and $c = 5$, the value of the expression $\dfrac{a^2 - b}{c^2 - b^2}$ is

(a) $\dfrac{1}{3}$ **(b)** $\dfrac{7}{34}$ **(c)** $\dfrac{7}{16}$ **(d)** $\dfrac{1}{16}$

(e) None of the above.

2. The expression $(-2a)\left(-4a^2b^3\right)^2$ simplifies to

(a) $16a^5b^6$ **(b)** $-32a^5b^5$ **(c)** $64a^6b^6$ **(d)** $-32a^5b^6$

(e) None of the above.

3. The expression $3x^2 + 5[4x^2 - 6(3x + 1)]$ simplifies to

(a) $23x^2 - 90x + 30$ **(b)** $23x^2 - 90x + 5$ **(c)** $23x^2 - 90x - 30$ **(d)** $23x^2 - 18x + 1$

(e) None of the above.

4. If $2(x + 3) + x = 6\left(1 - \dfrac{1}{3}x\right)$, then x is

(a) $\dfrac{3}{5}$ **(b)** $-\dfrac{3}{8}$ **(c)** 0 **(d)** $\dfrac{1}{5}$

(e) None of the above.

5. The expression $\dfrac{\sqrt[4]{x}\ \sqrt[3]{x}}{x}$ simplifies to

(a) $x^{-5/12}$ **(b)** x^{-8} **(c)** $x^{5/12}$ **(d)** $x^{19/12}$

(e) None of the above.

6. The quotient $\dfrac{(x+h)^2 - x^2}{h}$ simplifies to

(a) h **(b)** $2x + h$ **(c)** $x + h$ **(d)** $2xh + h$

(e) None of the above.

7. If $-5x + 1 < 5$, then

(a) $x > -\dfrac{4}{5}$ **(b)** $x < -\dfrac{4}{5}$ **(c)** $x > -\dfrac{6}{5}$ **(d)** $x < -\dfrac{5}{4}$

(e) None of the above.

8. If $\dfrac{5}{3x-4} = \dfrac{1}{x-2}$, then x is

(a) -1 **(b)** 3 **(c)** $\dfrac{3}{4}$ **(d)** 1

(e) None of the above.

9. The quadratic expression $x^2 - 7x + 10$ factors as

(a) $(x+2)(x+5)$ **(b)** $(x-2)(x-5)$ **(c)** $(x+1)(x+10)$ **(d)** $(x-1)(x-10)$

(e) None of the above.

10. The expression $\dfrac{3x-4}{x^2-5x+6} - \dfrac{1}{x-3}$ simplifies to

(a) $\dfrac{2(x-1)}{(x-2)(x-3)}$ **(b)** $\dfrac{2}{(x-2)}$ **(c)** $\dfrac{2(x-5)}{(x+2)(x-3)}$ **(d)** $\dfrac{2(2x-5)}{(x-2)(x-3)}$

(e) None of the above.

11. The polynomial $x^3 + 7x^2 + 6x$ can be factored as

(a) $x(x-1)(x-6)$ **(b)** $x(x-2)(x-3)$ **(c)** $x(x+2)(x+3)$

(d) $x(x+1)(x+6)$ **(e)** None of the above.

12. The expression $\dfrac{x^2-16}{x+4} \cdot \dfrac{3x-12}{x^2-8x+16}$ simplifies to

(a) 1 **(b)** 0 **(c)** 3 **(d)** $-\dfrac{3}{8x}$

(e) None of the above.

13. The quadratic equation $x^2 - x - 5 = 0$ is satisfied when x is

(a) $1 \pm \sqrt{6}$ **(b)** $\dfrac{1}{2}(1 \pm \sqrt{21})$ **(c)** $-\dfrac{1}{2}(1 \pm \sqrt{21})$ **(d)** $\dfrac{1}{2}(1 \pm \sqrt{6})$

(e) None of the above.

14. If $|x-2| \leq 5$, then

(a) $3 \leq x \leq 7$ **(b)** $-7 \leq x \leq -3$ **(c)** $-3 \leq x \leq 7$ **(d)** $-7 \leq x \leq 3$

(e) None of the above.

15. One factor of $9y^6 - 25x^2$ is

(a) $3y^3 + 5x$ **(b)** $9y^4 - 5x$ **(c)** $y^3 - 5x$ **(d)** $3y^4 + 25x$

(e) None of the above.

16. The inequality $\dfrac{x-2}{x+5} > 0$ is satisfied when

(a) $x < -5$ only **(b)** $x < -5$ or $x > 2$ **(c)** $x < 2$ or $x > 5$ **(d)** $x > 2$ only

(e) None of the above.

17. The expression $\left(\dfrac{x^{12}y^{-3}}{z^{-3}}\right)^{-\frac{4}{3}}$ is equivalent to

(a) $x^{-16}y^{-4}z^4$ **(b)** $x^{-16}y^4z^4$ **(c)** $x^{16}y^4z^{-4}$ **(d)** $x^{-16}y^4z^{-4}$

(e) None of the above.

18. The slope of the line with equation $3x + 4 = y - 5$ is

(a) $\frac{3}{5}$ **(b)** 3 **(c)** $\frac{3}{4}$ **(d)** $-\frac{3}{5}$

(e) None of the above.

19. A line with slope 3 and y-intercept 5 has the equation

(a) $y = 5x + 3$ **(b)** $x = 5y + 3$ **(c)** $y = 3x + 5$ **(d)** $x = 3y + 5$

(e) None of the above.

20. If (x,y) satisfies both of the equations $3x + 4y = 9$ and $3x - 4y = 6$, then x is

(a) $\frac{5}{2}$ **(b)** $\frac{1}{2}$ **(c)** $\frac{3}{2}$ **(d)** $\frac{9}{2}$

(e) None of the above.

21. If $f(x) = 2 - 4x$, then $f(x-2) + f(x) - 2$ is

(a) $14 - 8x$ **(b)** $10 - 8x$ **(c)** $-8x$ **(d)** $10 + 8x$

(e) None of the above.

22. The largest set of real numbers in the domain of the function $f(x) = \frac{1}{\sqrt{27 - x^3}}$ is

(a) $x < -3$ **(b)** $x \leq -3$ **(c)** $x < 3$ **(d)** $x \leq 3$

(e) None of the above.

23. If $f(x) = 2x - 3$ and $g(x) = 8x^2$, then the composition $(f \circ g)(x) \equiv f(g(x))$ is

(a) $8x^2 - 3$ **(b)** $32x^2 - 96x - 72$ **(c)** $16x^2 - 3$ **(d)** $16x^3 - 24x^2$

(e) None of the above.

24. The equation $\dfrac{x^2-5x+6}{x-2}+x=3$ is satisfied when x is

(a) 3 only **(b)** 2 or 3 **(c)** 0 only **(d)** 2 or $\dfrac{3}{2}$

(e) None of the above.

25. The expression $\dfrac{(2-x)(x+3)+(x+3)(x-4)}{(x+3)(2-x)}$ simplifies to

(a) x^2-x-11 **(b)** x^2-x-12 **(c)** $\dfrac{2}{x-2}$ **(d)** $x-3$

(e) None of the above.

26. The distance between the points $P(4,7)$ and $Q(-1,-5)$ is

(a) 5 **(b)** 17 **(c)** 13 **(d)** $\sqrt{13}$

(e) None of the above.

27. For positive real numbers m, n, and r, which of the following are true?

I. $\log(mn)=(\log m)(\log n)$

II. $\log\left(\dfrac{m}{n}\right)=\log m-\log n$

III. $\log\left(m^r\right)=r\log m$

(a) I, II, and III **(b)** I and II **(c)** I and III **(d)** II and III

(e) None of the above.

28. Suppose that 2^{11} is approximately $2,000$, which of the following best approximates 2^2

(a) $(4,000)^{11}$ **(b)** $40,000$ **(c)** $4,000,000$ **(d)** $(2,000)^{11}$

(e) None of the above.

29. A rectangle has a length that is 2 meters more than its width. What is the width of the rectangle if the perimeter of the rectangle is 52 meters?

(a) $-1+\sqrt{53}$ meters (b) 25 meters (c) 14 meters (d) 12 meters

(e) None of the above.

30. The volume of a sphere is proportional to the cube of its radius, and the surface area of a sphere is proportional to the square of the radius. Suppose the sphere S has a surface area that is 4 times the surface area of C. If the volume of C is 27, then the volume of S is

(a) 108 (b) 162 (c) 216 (d) 243

(e) None of the above.

31. The graph of $y = \sin\frac{1}{3}x$, for x in the interval $[0, 3\pi]$, crosses the x-axis at

(a) $0, \frac{3\pi}{2}$, and 3π (b) 0 and 3π (c) $\frac{3\pi}{2}$ only (d) 0 only

(e) None of the above.

32. When the expression $\sin t(\tan t + \cot t)$ is defined, it is equivalent to

(a) 1 (b) $\frac{1}{\sin t}$ (c) $\frac{1}{\cos t}$ (d) $(\sin t)^2 \cos t$

(e) None of the above.

33. The solutions of the equation $2\cos x - \sqrt{2} = 0$ that lie in the interval $[0, 2\pi]$ are

(a) $\frac{\pi}{4}$ and $\frac{3\pi}{4}$ (b) $\frac{\pi}{4}$ and $\frac{5\pi}{4}$ (c) $\frac{\pi}{4}$ and $\frac{7\pi}{4}$ (d) $\frac{3\pi}{4}$ and $\frac{7\pi}{4}$

(e) None of the above.

34. The value of $\tan(t - \pi)$ is the same as the value of

(a) $\tan t$ (b) $-\tan t$ (c) $\cot t$ (d) $-\cot t$

(e) None of the above.

35. An open rectangular box has height 3 and a square base. The volume of the box is 48. The surface area of the box is

(a) 32 (b) 64 (c) 80 (d) $24\sqrt[3]{6} + 8\sqrt[3]{36}$

(e) None of the above.

36. In an isosceles right triangle two sides have length 7. The length of the hypotenuse is

(a) $7\sqrt{2}$ (b) $\frac{7\sqrt{3}}{2}$ (c) $\sqrt{14}$ (d) $7\sqrt{3}$

(e) None of the above.

37. An angle of $\frac{8\pi}{3}$ radians is the same as an angle of

(a) 480° (b) 420° (c) 240° (d) 60°

(e) None of the above.

38. A square R has perimeter 8. A new square S is formed from R by multiplying all the sides of R by 3. How much more area does S have than R?

(a) 8 (b) 32 (c) 36 (d) 512

(e) None of the above.

39. The equation $x^2 + y^2 - 6x - 2y - 15 = 0$ describes a circle with

(a) center $(-3, -1)$ and radius 5 (b) center $(3, 1)$ and radius 5

(c) center $(-3, -1)$ and radius 25 (d) center $(3, 1)$ and radius 25

(e) None of the above.

40. The graph of the equation $y = \frac{3x}{x^2 - 9}$ has a vertical asymptote whose equation is

(a) $x = 3$ (b) $y = 3$ (c) $x = 9$ (d) $y = -3$

(e) None of the above.